6th Form Transition Pack

| QUALIFICATION	A-level Maths
Teacher Name(s)	Mr Cameron Miss Munro
Contact email(s)	lcameron3@notredame.liverpool.sch.uk
Exam board and link	Edexcel (Pearson) www.pearson.co.uk
Specification details	9MA0
Recommended online learning	https://www.drfrostmaths.com/ https://madasmaths.com/ https://www.khanacademy.org/ http://mathsgenie.co.uk/ http://physicsandmathstutor.com/
Textbooks that are used	

AS and A2 MATHEMATICS
Edexcel
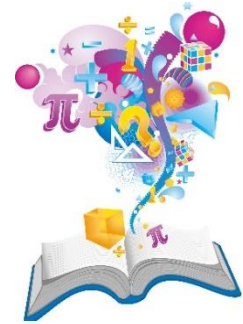
Why Study Maths?

There are many reasons why people choose to study A Level Mathematics. It might be a requirement for what you want to study at university (Physics, Psychology, Economics, Computing, and Business studies prefer students to have A Level Maths if possible). Since Maths is one of the most traditional subjects a good grade in Maths can boost an application for pretty much every course!
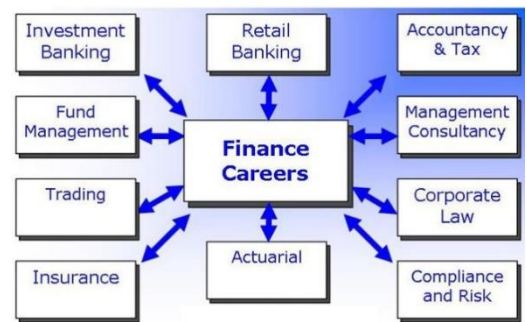
Studies have also shown that people with Maths A Level also tend to earn more on average than people without it. Though this itself may or may not be a good enough reason to study Maths, the skills it allows you to develop include problem solving, logic and analysing situations. Add in the improvements to your basic numeracy skills and the creativity needed to solve Maths problems and you've got yourself a set of skills which would make you more desirable for almost any job!

Finally, you might also really like Maths - this is as good a reason as any to continue studying it. If you study something you enjoy you are likely to do better at it. With Maths there is the excitement of new discoveries you will make. You will see more of the beauty of it and realise just how much everything around in the universe is connected to mathematics.

Maths is an amazing subject to have at A Level and provided you have a solid understanding of the GCSE concepts before you start, alongside some perseverance and effort, you should be able to do well.

What will you study?

In year 12 you will study Pure Maths Mechanics and Statistics building the foundations for the year 13 content.

In year 13 you will study additional content in Pure Maths Statistics and Mechanics.

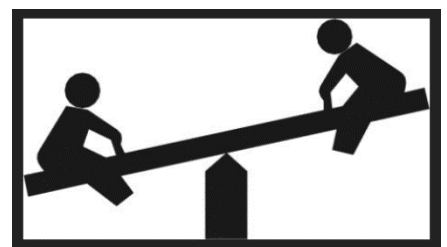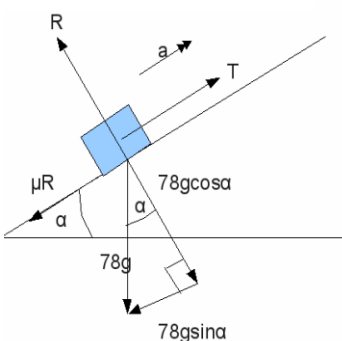The **Pure Content** is algebra, calculus and geometry based.
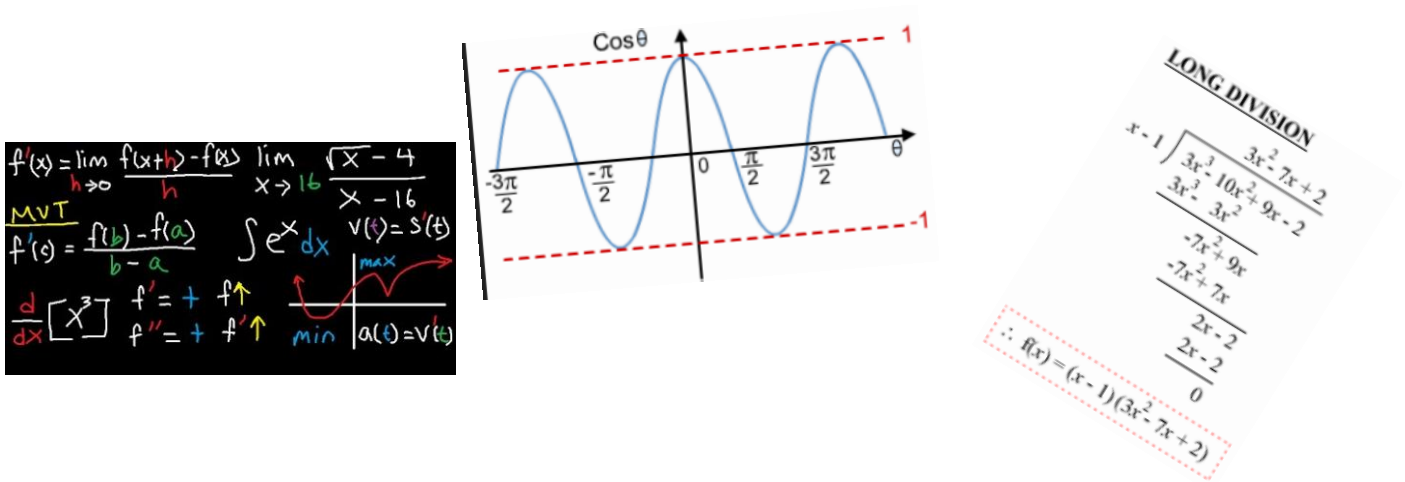
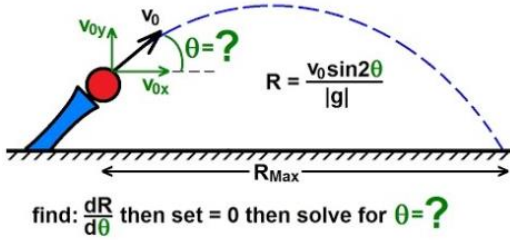**Statistics** involves analysing data with formulae and looking into the probability of why things happen.

$$r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}$$
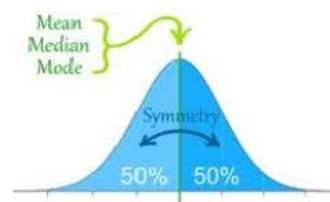
$$P(x) = \binom{n}{x} p^x q^{n-x} = \frac{n!}{(n-x)!\,x!} p^x q^{n-x}$$

Mechanics looks at how and why objects move in the way they do.

find: $\frac{dR}{d\theta}$ then set = 0 then solve for θ = ?

Assessment

The new specification of the A level Maths is now all covered in three exams at the end of year 13.

Paper 1: Pure Mathematics 1 (*Paper code: 9MA0/01)
Paper 2: Pure Mathematics 2 (*Paper code: 9MA0/02)
Each paper is:
2-hour written examination
33.33% of the qualification
100 marks
Content overview
• Topic 1 – Proof
• Topic 2 – Algebra and functions
• Topic 3 – Coordinate geometry in the (x, y) plane
• Topic 4 – Sequences and series
• Topic 5 – Trigonometry
• Topic 6 – Exponentials and logarithms
• Topic 7 – Differentiation
• Topic 8 – Integration
• Topic 9 – Numerical methods
• Topic 10 – Vectors
Assessment overview
• Paper 1 and Paper 2 may contain questions on any topics from the Pure Mathematics content.
• Students must answer all questions.
• Calculators can be used in the assessment.

Paper 3: Statistics and Mechanics (*Paper code: 9MA0/03)
2-hour written examination
33.33% of the qualification
100 marks
Content overview
Section A: Statistics
• Topic 1 – Statistical sampling
• Topic 2 – Data presentation and interpretation
• Topic 3 – Probability
• Topic 4 – Statistical distributions
• Topic 5 – Statistical hypothesis testing
Section B: Mechanics
• Topic 6 – Quantities and units in mechanics
• Topic 7 – Kinematics
• Topic 8 – Forces and Newton's laws
• Topic 9 – Moments
Assessment overview
• Paper 3 will contain questions on topics from the Statistics content in Section A and Mechanics content in Section B.
• Students must answer all questions.
• Calculators can be used in the assessment.

At Notre Dame Catholic College, we study the Edexcel specification
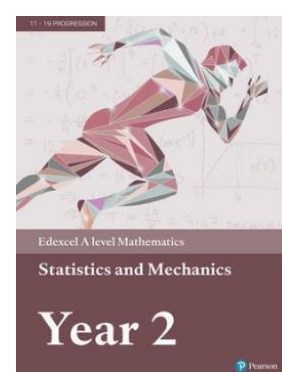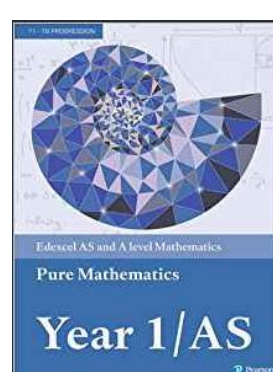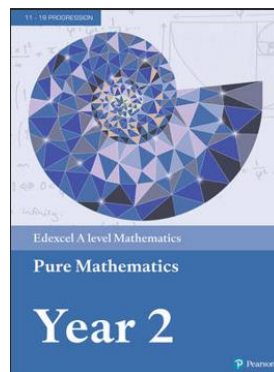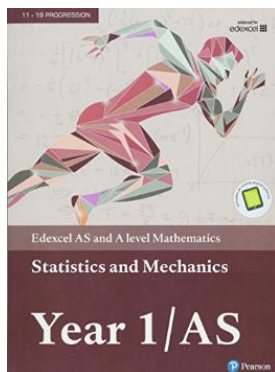
Expanding brackets and simplifying expressions

A LEVEL LINKS
Scheme of work: 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

- When you expand one set of brackets you must multiply everything inside the bracket by what is outside.
- When you expand two linear expressions, each with two terms of the form $ax + b$, where $a \neq 0$ and $b \neq 0$, you create four terms. Two of these can usually be simplified by collecting like terms.

Examples

Example 1   Expand $4(3x - 2)$

$4(3x - 2) = 12x - 8$	Multiply everything inside the bracket by the 4 outside the bracket

Example 2   Expand and simplify $3(x + 5) - 4(2x + 3)$

$3(x + 5) - 4(2x + 3)$ $= 3x + 15 - 8x - 12$ $= 3 - 5x$	**1** Expand each set of brackets separately by multiplying $(x + 5)$ by 3 and $(2x + 3)$ by -4 **2** Simplify by collecting like terms: $3x - 8x = -5x$ and $15 - 12 = 3$

Example 3   Expand and simplify $(x + 3)(x + 2)$

$(x + 3)(x + 2)$ $= x(x + 2) + 3(x + 2)$ $= x^2 + 2x + 3x + 6$ $= x^2 + 5x + 6$	**1** Expand the brackets by multiplying $(x + 2)$ by x and $(x + 2)$ by 3 **2** Simplify by collecting like terms: $2x + 3x = 5x$

Example 4   Expand and simplify $(x - 5)(2x + 3)$

$(x - 5)(2x + 3)$ $= x(2x + 3) - 5(2x + 3)$ $= 2x^2 + 3x - 10x - 15$ $= 2x^2 - 7x - 15$	**1** Expand the brackets by multiplying $(2x + 3)$ by x and $(2x + 3)$ by -5 **2** Simplify by collecting like terms: $3x - 10x = -7x$

Practice

1   Expand.

> **Watch out!**
>
> When multiplying (or dividing) positive and negative numbers, if the signs are the same the answer is '+'; if the signs are different the answer is '–'.

a $3(2x - 1)$ **b** $-2(5pq + 4q^2)$

c $-(3xy - 2y^2)$

2 Expand and simplify.

    a $7(3x + 5) + 6(2x - 8)$ **b** $8(5p - 2) - 3(4p + 9)$

    c $9(3s + 1) - 5(6s - 10)$ **d** $2(4x - 3) - (3x + 5)$

3 Expand.

    a $3x(4x + 8)$ **b** $4k(5k^2 - 12)$

    c $-2h(6h^2 + 11h - 5)$ **d** $-3s(4s^2 - 7s + 2)$

4 Expand and simplify.

    a $3(y^2 - 8) - 4(y^2 - 5)$ **b** $2x(x + 5) + 3x(x - 7)$

    c $4p(2p - 1) - 3p(5p - 2)$ **d** $3b(4b - 3) - b(6b - 9)$

5 Expand $\frac{1}{2}(2y - 8)$

6 Expand and simplify.

    a $13 - 2(m + 7)$ **b** $5p(p^2 + 6p) - 9p(2p - 3)$

7 The diagram shows a rectangle.

Write down an expression, in terms of x, for the area of the rectangle.
Show that the area of the rectangle can be written as $21x^2 - 35x$
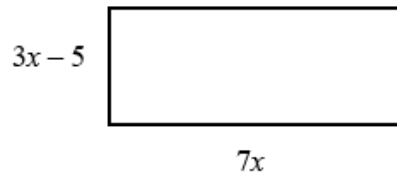
$3x - 5$ [rectangle] $7x$

8 Expand and simplify.

    a $(x + 4)(x + 5)$ **b** $(x + 7)(x + 3)$

    c $(x + 7)(x - 2)$ **d** $(x + 5)(x - 5)$

    e $(2x + 3)(x - 1)$ **f** $(3x - 2)(2x + 1)$

    g $(5x - 3)(2x - 5)$ **h** $(3x - 2)(7 + 4x)$

    i $(3x + 4y)(5y + 6x)$ **j** $(x + 5)^2$

    k $(2x - 7)^2$ **l** $(4x - 3y)^2$

Extend

9 Expand and simplify $(x + 3)^2 + (x - 4)^2$

10 Expand and simplify.

    a $\left(x + \dfrac{1}{x}\right)\left(x - \dfrac{2}{x}\right)$ **b** $\left(x + \dfrac{1}{x}\right)^2$

Surds and rationalising the denominator

Key points

- A surd is the square root of a number that is not a square number,
  for example $\sqrt{2}, \sqrt{3}, \sqrt{5},$ etc.
- Surds can be used to give the exact value for an answer.
- $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- $\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}$
- To rationalise the denominator means to remove the surd from the denominator of a fraction.
- To rationalise $\dfrac{a}{\sqrt{b}}$ you multiply the numerator and denominator by the surd $\sqrt{b}$
- To rationalise $\dfrac{a}{b+\sqrt{c}}$ you multiply the numerator and denominator by $b-\sqrt{c}$

Examples

Example 1   Simplify $\sqrt{50}$

$\sqrt{50} = \sqrt{25 \times 2}$	**1** Choose two numbers that are factors of 50. One of the factors must be a square number
$= \sqrt{25} \times \sqrt{2}$	**2** Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
$= 5 \times \sqrt{2}$	**3** Use $\sqrt{25} = 5$
$= 5\sqrt{2}$	

Example 2   Simplify $\sqrt{147} - 2\sqrt{12}$

$\sqrt{147} - 2\sqrt{12}$ $= \sqrt{49 \times 3} - 2\sqrt{4 \times 3}$	**1** Simplify $\sqrt{147}$ and $2\sqrt{12}$. Choose two numbers that are factors of 147 and two numbers that are factors of 12. One of each pair of factors must be a square number
$= \sqrt{49} \times \sqrt{3} - 2\sqrt{4} \times \sqrt{3}$	**2** Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
$= 7 \times \sqrt{3} - 2 \times 2 \times \sqrt{3}$	**3** Use $\sqrt{49} = 7$ and $\sqrt{4} = 2$
$= 7\sqrt{3} - 4\sqrt{3}$	
$= 3\sqrt{3}$	**4** Collect like terms

Example 3   Simplify $\left(\sqrt{7}+\sqrt{2}\right)\left(\sqrt{7}-\sqrt{2}\right)$

$\left(\sqrt{7}+\sqrt{2}\right)\left(\sqrt{7}-\sqrt{2}\right)$ $=\sqrt{49}-\sqrt{7}\sqrt{2}+\sqrt{2}\sqrt{7}-\sqrt{4}$ $=7-2$ $=5$	**1** Expand the brackets. A common mistake here is to write $\left(\sqrt{7}\right)^2=49$ **2** Collect like terms: $-\sqrt{7}\sqrt{2}+\sqrt{2}\sqrt{7}$ $\qquad =-\sqrt{7}\sqrt{2}+\sqrt{7}\sqrt{2}=0$

Example 4   Rationalise $\dfrac{1}{\sqrt{3}}$

$\dfrac{1}{\sqrt{3}}=\dfrac{1}{\sqrt{3}}\times\dfrac{\sqrt{3}}{\sqrt{3}}$ $=\dfrac{1\times\sqrt{3}}{\sqrt{9}}$ $=\dfrac{\sqrt{3}}{3}$	**1** Multiply the numerator and denominator by $\sqrt{3}$ **2** Use $\sqrt{9}=3$

Example 5   Rationalise and simplify $\dfrac{\sqrt{2}}{\sqrt{12}}$

$\dfrac{\sqrt{2}}{\sqrt{12}}=\dfrac{\sqrt{2}}{\sqrt{12}}\times\dfrac{\sqrt{12}}{\sqrt{12}}$ $=\dfrac{\sqrt{2}\times\sqrt{4\times3}}{12}$ $=\dfrac{2\sqrt{2}\sqrt{3}}{12}$ $=\dfrac{\sqrt{2}\sqrt{3}}{6}$	**1** Multiply the numerator and denominator by $\sqrt{12}$ **2** Simplify $\sqrt{12}$ in the numerator. Choose two numbers that are factors of 12. One of the factors must be a square number **3** Use the rule $\sqrt{ab}=\sqrt{a}\times\sqrt{b}$ **4** Use $\sqrt{4}=2$ **5** Simplify the fraction: $\dfrac{2}{12}$ simplifies to $\dfrac{1}{6}$

Example 6 Rationalise and simplify $\dfrac{3}{2+\sqrt{5}}$

$\dfrac{3}{2+\sqrt{5}} = \dfrac{3}{2+\sqrt{5}} \times \dfrac{2-\sqrt{5}}{2-\sqrt{5}}$	**1** Multiply the numerator and denominator by $2-\sqrt{5}$
$= \dfrac{3\left(2-\sqrt{5}\right)}{\left(2+\sqrt{5}\right)\left(2-\sqrt{5}\right)}$	**2** Expand the brackets
$= \dfrac{6-3\sqrt{5}}{4+2\sqrt{5}-2\sqrt{5}-5}$	**3** Simplify the fraction
$= \dfrac{6-3\sqrt{5}}{-1}$	**4** Divide the numerator by -1 Remember to change the sign of all terms when dividing by -1
$= 3\sqrt{5}-6$	

Practice

1 Simplify.

  a $\sqrt{45}$ **b** $\sqrt{125}$

  c $\sqrt{48}$ **d** $\sqrt{175}$

  e $\sqrt{300}$ **f** $\sqrt{28}$

  g $\sqrt{72}$ **h** $\sqrt{162}$

> **Hint**
>
> One of the two numbers you choose at the start must be a square number.

2 Simplify.

  a $\sqrt{72}+\sqrt{162}$ **b** $\sqrt{45}-2\sqrt{5}$

  c $\sqrt{50}-\sqrt{8}$ **d** $\sqrt{75}-\sqrt{48}$

  e $2\sqrt{28}+\sqrt{28}$ **f** $2\sqrt{12}-\sqrt{12}+\sqrt{27}$

> **Watch out!**
>
> Check you have chosen the highest square number at the

3 Expand and simplify.

  a $(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})$ **b** $(3+\sqrt{3})(5-\sqrt{12})$

  c $(4-\sqrt{5})(\sqrt{45}+2)$ **d** $(5+\sqrt{2})(6-\sqrt{8})$

4 Rationalise and simplify, if possible.

    a $\dfrac{1}{\sqrt{5}}$ **b** $\dfrac{1}{\sqrt{11}}$

    c $\dfrac{2}{\sqrt{7}}$ **d** $\dfrac{2}{\sqrt{8}}$

    e $\dfrac{2}{\sqrt{2}}$ **f** $\dfrac{5}{\sqrt{5}}$

    g $\dfrac{\sqrt{8}}{\sqrt{24}}$ **h** $\dfrac{\sqrt{5}}{\sqrt{45}}$

5 Rationalise and simplify.

    a $\dfrac{1}{3-\sqrt{5}}$ **b** $\dfrac{2}{4+\sqrt{3}}$ **c** $\dfrac{6}{5-\sqrt{2}}$

Extend

6 Expand and simplify $\left(\sqrt{x}+\sqrt{y}\right)\left(\sqrt{x}-\sqrt{y}\right)$

7 Rationalise and simplify, if possible.

    a $\dfrac{1}{\sqrt{9}-\sqrt{8}}$ **b** $\dfrac{1}{\sqrt{x}-\sqrt{y}}$

Rules of indices

A LEVEL LINKS
Scheme of work: 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

- $a^m \times a^n = a^{m+n}$
- $\dfrac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^0 = 1$
- $a^{\frac{1}{n}} = \sqrt[n]{a}$ i.e. the nth root of a
- $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$
- $a^{-m} = \dfrac{1}{a^m}$
- The square root of a number produces two solutions, e.g. $\sqrt{16} = \pm 4$.

Examples

Example 1   Evaluate 10^0

$10^0 = 1$	Any value raised to the power of zero is equal to 1

Example 2   Evaluate $9^{\frac{1}{2}}$

$9^{\frac{1}{2}} = \sqrt{9}$ $\quad = 3$	Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$

Example 3   Evaluate $27^{\frac{2}{3}}$

$27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2$ $\quad = 3^2$ $\quad = 9$	**1** Use the rule $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$ **2** Use $\sqrt[3]{27} = 3$

Example 4   Evaluate 4^{-2}

$4^{-2} = \dfrac{1}{4^2}$	**1** Use the rule $a^{-m} = \dfrac{1}{a^m}$
$= \dfrac{1}{16}$	**2** Use $4^2 = 16$

Example 5   Simplify $\dfrac{6x^5}{2x^2}$

$\dfrac{6x^5}{2x^2} = 3x^3$	$6 \div 2 = 3$ and use the rule $\dfrac{a^m}{a^n} = a^{m-n}$ to give $\dfrac{x^5}{x^2} = x^{5-2} = x^3$

Example 6   Simplify $\dfrac{x^3 \times x^5}{x^4}$

$\dfrac{x^3 \times x^5}{x^4} = \dfrac{x^{3+5}}{x^4} = \dfrac{x^8}{x^4}$	**1** Use the rule $a^m \times a^n = a^{m+n}$
$= x^{8-4} = x^4$	**2** Use the rule $\dfrac{a^m}{a^n} = a^{m-n}$

Example 7   Write $\dfrac{1}{3x}$ as a single power of x

$\dfrac{1}{3x} = \dfrac{1}{3} x^{-1}$	Use the rule $\dfrac{1}{a^m} = a^{-m}$, note that the fraction $\dfrac{1}{3}$ remains unchanged

Example 8   Write $\dfrac{4}{\sqrt{x}}$ as a single power of x

$\dfrac{4}{\sqrt{x}} = \dfrac{4}{x^{\frac{1}{2}}}$	**1** Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$
$= 4x^{-\frac{1}{2}}$	**2** Use the rule $\dfrac{1}{a^m} = a^{-m}$

Practice

1 Evaluate.

a 14^0 **b** 3^0 **c** 5^0 **d** x^0

2 Evaluate.

a $49^{\frac{1}{2}}$ **b** $64^{\frac{1}{3}}$ **c** $125^{\frac{1}{3}}$ **d** $16^{\frac{1}{4}}$

3 Evaluate.

a $25^{\frac{3}{2}}$ **b** $8^{\frac{5}{3}}$ **c** $49^{\frac{3}{2}}$ **d** $16^{\frac{3}{4}}$

4 Evaluate.

a 5^{-2} **b** 4^{-3} **c** 2^{-5} **d** 6^{-2}

5 Simplify.

a $\dfrac{3x^2 \times x^3}{2x^2}$ **b** $\dfrac{10x^5}{2x^2 \times x}$

c $\dfrac{3x \times 2x^3}{2x^3}$ **d** $\dfrac{7x^3 y^2}{14x^5 y}$

e $\dfrac{y^2}{y^{\frac{1}{2}} \times y}$ **f** $\dfrac{c^{\frac{1}{2}}}{c^2 \times c^{\frac{3}{2}}}$

g $\dfrac{\left(2x^2\right)^3}{4x^0}$ **h** $\dfrac{x^{\frac{1}{2}} \times x^{\frac{3}{2}}}{x^{-2} \times x^3}$

> **Watch out!**
>
> Remember that any value raised to the power of zero is 1. This is the rule $a^0 = 1$.

6 Evaluate.

a $4^{-\frac{1}{2}}$ **b** $27^{-\frac{2}{3}}$ **c** $9^{-\frac{1}{2}} \times 2^3$

d $16^{\frac{1}{4}} \times 2^{-3}$ **e** $\left(\dfrac{9}{16}\right)^{-\frac{1}{2}}$ **f** $\left(\dfrac{27}{64}\right)^{-\frac{2}{3}}$

7 Write the following as a single power of x.

a $\dfrac{1}{x}$ **b** $\dfrac{1}{x^7}$ **c** $\sqrt[4]{x}$

d $\sqrt[5]{x^2}$ **e** $\dfrac{1}{\sqrt[3]{x}}$ **f** $\dfrac{1}{\sqrt[3]{x^2}}$

8   Write the following without negative or fractional powers.

a x^{-3}

b x^0

c $x^{\frac{1}{5}}$

d $x^{\frac{2}{5}}$

e $x^{-\frac{1}{2}}$

f $x^{-\frac{3}{4}}$

9   Write the following in the form ax^n.

a $5\sqrt{x}$

b $\dfrac{2}{x^3}$

c $\dfrac{1}{3x^4}$

d $\dfrac{2}{\sqrt{x}}$

e $\dfrac{4}{\sqrt[3]{x}}$

f 3

Extend

10   Write as sums of powers of x.

a $\dfrac{x^5+1}{x^2}$

b $x^2\left(x+\dfrac{1}{x}\right)$

c $x^{-4}\left(x^2+\dfrac{1}{x^3}\right)$

Factorising expressions

Key points

- Factorising an expression is the opposite of expanding the brackets.
- A quadratic expression is in the form $ax^2 + bx + c$, where $a \neq 0$.
- To factorise a quadratic equation find two numbers whose sum is b and whose product is ac.
- An expression in the form $x^2 - y^2$ is called the difference of two squares. It factorises to $(x - y)(x + y)$.

Examples

Example 1   Factorise $15x^2y^3 + 9x^4y$

$15x^2y^3 + 9x^4y = 3x^2y(5y^2 + 3x^2)$	The highest common factor is $3x^2y$. So take $3x^2y$ outside the brackets and then divide each term by $3x^2y$ to find the terms in the brackets

Example 2   Factorise $4x^2 - 25y^2$

$4x^2 - 25y^2 = (2x + 5y)(2x - 5y)$	This is the difference of two squares as the two terms can be written as $(2x)^2$ and $(5y)^2$

Example 3   Factorise $x^2 + 3x - 10$

$b = 3$, $ac = -10$	**1** Work out the two factors of $ac = -10$ which add to give $b = 3$ (5 and −2)
So $x^2 + 3x - 10 = x^2 + 5x - 2x - 10$	**2** Rewrite the b term $(3x)$ using these two factors
$\quad = x(x + 5) - 2(x + 5)$	**3** Factorise the first two terms and the last two terms
$\quad = (x + 5)(x - 2)$	**4** $(x + 5)$ is a factor of both terms

Example 4   Factorise $6x^2 - 11x - 10$

$b = -11$, $ac = -60$	**1** Work out the two factors of $ac = -60$ which add to give $b = -11$ (−15 and 4)
So $6x^2 - 11x - 10 = 6x^2 - 15x + 4x - 10$	**2** Rewrite the b term ($-11x$) using these two factors
$= 3x(2x - 5) + 2(2x - 5)$	**3** Factorise the first two terms and the last two terms
$= (2x - 5)(3x + 2)$	**4** $(2x - 5)$ is a factor of both terms

Example 5   Simplify $\dfrac{x^2 - 4x - 21}{2x^2 + 9x + 9}$

$\dfrac{x^2 - 4x - 21}{2x^2 + 9x + 9}$	**1** Factorise the numerator and the denominator
For the numerator: $b = -4$, $ac = -21$	**2** Work out the two factors of $ac = -21$ which add to give $b = -4$ (−7 and 3)
So $x^2 - 4x - 21 = x^2 - 7x + 3x - 21$	**3** Rewrite the b term ($-4x$) using these two factors
$= x(x - 7) + 3(x - 7)$	**4** Factorise the first two terms and the last two terms
$= (x - 7)(x + 3)$	**5** $(x - 7)$ is a factor of both terms
For the denominator: $b = 9$, $ac = 18$	**6** Work out the two factors of $ac = 18$ which add to give $b = 9$ (6 and 3)
So $2x^2 + 9x + 9 = 2x^2 + 6x + 3x + 9$	**7** Rewrite the b term ($9x$) using these two factors
$= 2x(x + 3) + 3(x + 3)$	**8** Factorise the first two terms and the last two terms
$= (x + 3)(2x + 3)$	**9** $(x + 3)$ is a factor of both terms
So $\dfrac{x^2 - 4x - 21}{2x^2 + 9x + 9} = \dfrac{(x - 7)(x + 3)}{(x + 3)(2x + 3)}$ $= \dfrac{x - 7}{2x + 3}$	**10** $(x + 3)$ is a factor of both the numerator and denominator so cancels out as a value divided by itself is 1

Practice

1 Factorise.

   a $6x^4y^3 - 10x^3y^4$ **b** $21a^3b^5 + 35a^5b^2$

   c $25x^2y^2 - 10x^3y^2 + 15x^2y^3$

> **Hint**
>
> Take the highest common factor outside the bracket.

2 Factorise

   a $x^2 + 7x + 12$ **b** $x^2 + 5x - 14$

   c $x^2 - 11x + 30$ **d** $x^2 - 5x - 24$

   e $x^2 - 7x - 18$ **f** $x^2 + x - 20$

   g $x^2 - 3x - 40$ **h** $x^2 + 3x - 28$

3 Factorise

   a $36x^2 - 49y^2$ **b** $4x^2 - 81y^2$

   c $18a^2 - 200b^2c^2$

4 Factorise

   a $2x^2 + x - 3$ **b** $6x^2 + 17x + 5$

   c $2x^2 + 7x + 3$ **d** $9x^2 - 15x + 4$

   e $10x^2 + 21x + 9$ **f** $12x^2 - 38x + 20$

5 Simplify the algebraic fractions.

   a $\dfrac{2x^2 + 4x}{x^2 - x}$ **b** $\dfrac{x^2 + 3x}{x^2 + 2x - 3}$

   c $\dfrac{x^2 - 2x - 8}{x^2 - 4x}$ **d** $\dfrac{x^2 - 5x}{x^2 - 25}$

   e $\dfrac{x^2 - x - 12}{x^2 - 4x}$ **f** $\dfrac{2x^2 + 14x}{2x^2 + 4x - 70}$

6 Simplify

   a $\dfrac{9x^2 - 16}{3x^2 + 17x - 28}$ **b** $\dfrac{2x^2 - 7x - 15}{3x^2 - 17x + 10}$

   c $\dfrac{4 - 25x^2}{10x^2 - 11x - 6}$ **d** $\dfrac{6x^2 - x - 1}{2x^2 + 7x - 4}$

Extend

7 Simplify $\sqrt{x^2 + 10x + 25}$

8   Simplify $\dfrac{(x+2)^2 + 3(x+2)^2}{x^2 - 4}$

Completing the square

Key points

- Completing the square for a quadratic rearranges $ax^2 + bx + c$ into the form $p(x + q)^2 + r$
- If $a \neq 1$, then factorise using a as a common factor.

Examples

Example 1 Complete the square for the quadratic expression $x^2 + 6x - 2$

$x^2 + 6x - 2$ $= (x + 3)^2 - 9 - 2$ $= (x + 3)^2 - 11$	**1** Write $x^2 + bx + c$ in the form $\left(x + \dfrac{b}{2}\right)^2 - \left(\dfrac{b}{2}\right)^2 + c$ **2** Simplify

Example 2 Write $2x^2 - 5x + 1$ in the form $p(x + q)^2 + r$

$2x^2 - 5x + 1$ $= 2\left(x^2 - \dfrac{5}{2}x\right) + 1$ $= 2\left[\left(x - \dfrac{5}{4}\right)^2 - \left(\dfrac{5}{4}\right)^2\right] + 1$ $= 2\left(x - \dfrac{5}{4}\right)^2 - \dfrac{25}{8} + 1$ $= 2\left(x - \dfrac{5}{4}\right)^2 - \dfrac{17}{8}$	**1** Before completing the square write $ax^2 + bx + c$ in the form $a\left(x^2 + \dfrac{b}{a}x\right) + c$ **2** Now complete the square by writing $x^2 - \dfrac{5}{2}x$ in the form $\left(x + \dfrac{b}{2}\right)^2 - \left(\dfrac{b}{2}\right)^2$ **3** Expand the square brackets – don't forget to multiply $\left(\dfrac{5}{4}\right)^2$ by the factor of 2 **4** Simplify

Practice

1 Write the following quadratic expressions in the form $(x + p)^2 + q$

    a $x^2 + 4x + 3$ **b** $x^2 - 10x - 3$

    c $x^2 - 8x$ **d** $x^2 + 6x$

    e $x^2 - 2x + 7$ **f** $x^2 + 3x - 2$

2 Write the following quadratic expressions in the form $p(x + q)^2 + r$

    a $2x^2 - 8x - 16$ **b** $4x^2 - 8x - 16$

    c $3x^2 + 12x - 9$ **d** $2x^2 + 6x - 8$

3 Complete the square.

    a $2x^2 + 3x + 6$ **b** $3x^2 - 2x$

    c $5x^2 + 3x$ **d** $3x^2 + 5x + 3$

Extend

4 Write $(25x^2 + 30x + 12)$ in the form $(ax + b)^2 + c$.

Solving quadratic equations by factorisation

Key points

- A quadratic equation is an equation in the form $ax^2 + bx + c = 0$ where $a \neq 0$.
- To factorise a quadratic equation find two numbers whose sum is b and whose products is ac.
- When the product of two numbers is 0, then at least one of the numbers must be 0.
- If a quadratic can be solved it will have two solutions (these may be equal).

Examples

Example 1   Solve $5x^2 = 15x$

$5x^2 = 15x$	**1** Rearrange the equation so that all of the terms are on one side of the equation and it is equal to zero.
$5x^2 - 15x = 0$	Do not divide both sides by x as this would lose the solution $x = 0$.
$5x(x - 3) = 0$	**2** Factorise the quadratic equation. $5x$ is a common factor.
So $5x = 0$ or $(x - 3) = 0$	**3** When two values multiply to make zero, at least one of the values must be zero.
Therefore $x = 0$ or $x = 3$	**4** Solve these two equations.

Example 2   Solve $x^2 + 7x + 12 = 0$

$x^2 + 7x + 12 = 0$	**1** Factorise the quadratic equation. Work out the two factors of $ac = 12$ which add to give you $b = 7$.
$b = 7$, $ac = 12$	(4 and 3)
$x^2 + 4x + 3x + 12 = 0$	**2** Rewrite the b term ($7x$) using these two factors.
$x(x + 4) + 3(x + 4) = 0$	**3** Factorise the first two terms and the last two terms.
$(x + 4)(x + 3) = 0$	**4** $(x + 4)$ is a factor of both terms.
So $(x + 4) = 0$ or $(x + 3) = 0$	**5** When two values multiply to make zero, at least one of the values must be zero.
Therefore $x = -4$ or $x = -3$	**6** Solve these two equations.

Example 3   Solve $9x^2 - 16 = 0$

$9x^2 - 16 = 0$ $(3x + 4)(3x - 4) = 0$ So $(3x + 4) = 0$ or $(3x - 4) = 0$ $x = -\dfrac{4}{3}$ or $x = \dfrac{4}{3}$	**1** Factorise the quadratic equation. This is the difference of two squares as the two terms are $(3x)^2$ and $(4)^2$. **2** When two values multiply to make zero, at least one of the values must be zero. **3** Solve these two equations.

Example 4   Solve $2x^2 - 5x - 12 = 0$

$b = -5$, $ac = -24$ So $2x^2 - 8x + 3x - 12 = 0$ $2x(x - 4) + 3(x - 4) = 0$ $(x - 4)(2x + 3) = 0$ So $(x - 4) = 0$ or $(2x + 3) = 0$ $x = 4$ or $x = -\dfrac{3}{2}$	**1** Factorise the quadratic equation. Work out the two factors of $ac = -24$ which add to give you $b = -5$. $(-8$ and $3)$ **2** Rewrite the b term $(-5x)$ using these two factors. **3** Factorise the first two terms and the last two terms. **4** $(x - 4)$ is a factor of both terms. **5** When two values multiply to make zero, at least one of the values must be zero. **6** Solve these two equations.

Practice

1   Solve

   a $6x^2 + 4x = 0$ **b** $28x^2 - 21x = 0$

   c $x^2 + 7x + 10 = 0$ **d** $x^2 - 5x + 6 = 0$

   e $x^2 - 3x - 4 = 0$ **f** $x^2 + 3x - 10 = 0$

   g $x^2 - 10x + 24 = 0$ **h** $x^2 - 36 = 0$

   i $x^2 + 3x - 28 = 0$ **j** $x^2 - 6x + 9 = 0$

   k $2x^2 - 7x - 4 = 0$ **l** $3x^2 - 13x - 10 = 0$

2   Solve

   a $x^2 - 3x = 10$ **b** $x^2 - 3 = 2x$

   c $x^2 + 5x = 24$ **d** $x^2 - 42 = x$

   e $x(x + 2) = 2x + 25$ **f** $x^2 - 30 = 3x - 2$

   g $x(3x + 1) = x^2 + 15$ **h** $3x(x - 1) = 2(x + 1)$

> **Hint**
>
> Get all terms onto one side of the

Solving quadratic equations by completing the square

Key points

- Completing the square lets you write a quadratic equation in the form $p(x + q)^2 + r = 0$.

Examples

Example 5   Solve $x^2 + 6x + 4 = 0$. Give your solutions in surd form.

$x^2 + 6x + 4 = 0$ $(x + 3)^2 - 9 + 4 = 0$	**1** Write $x^2 + bx + c = 0$ in the form $$\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c = 0$$
$(x + 3)^2 - 5 = 0$ $(x + 3)^2 = 5$	**2** Simplify. **3** Rearrange the equation to work out x. First, add 5 to both sides.
$x + 3 = \pm\sqrt{5}$	**4** Square root both sides. Remember that the square root of a value gives two answers.
$x = \pm\sqrt{5} - 3$	**5** Subtract 3 from both sides to solve the equation.
So $x = -\sqrt{5} - 3$ or $x = \sqrt{5} - 3$	**6** Write down both solutions.

Example 6   Solve $2x^2 - 7x + 4 = 0$. Give your solutions in surd form.

$2x^2 - 7x + 4 = 0$ $2\left(x^2 - \dfrac{7}{2}x\right) + 4 = 0$	**1** Before completing the square write $ax^2 + bx + c$ in the form $$a\left(x^2 + \frac{b}{a}x\right) + c$$
$2\left[\left(x - \dfrac{7}{4}\right)^2 - \left(\dfrac{7}{4}\right)^2\right] + 4 = 0$	**2** Now complete the square by writing $x^2 - \dfrac{7}{2}x$ in the form $$\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2$$
$2\left(x - \dfrac{7}{4}\right)^2 - \dfrac{49}{8} + 4 = 0$	**3** Expand the square brackets.
$2\left(x - \dfrac{7}{4}\right)^2 - \dfrac{17}{8} = 0$	**4** Simplify.

$2\left(x-\dfrac{7}{4}\right)^2=\dfrac{17}{8}$	**5** Rearrange the equation to work out x. First, add $\dfrac{17}{8}$ to both sides.
$\left(x-\dfrac{7}{4}\right)^2=\dfrac{17}{16}$	**6** Divide both sides by 2.
$x-\dfrac{7}{4}=\pm\dfrac{\sqrt{17}}{4}$	**7** Square root both sides. Remember that the square root of a value gives two answers.
$x=\pm\dfrac{\sqrt{17}}{4}+\dfrac{7}{4}$	**8** Add $\dfrac{7}{4}$ to both sides.
So $x=\dfrac{7}{4}-\dfrac{\sqrt{17}}{4}$ or $x=\dfrac{7}{4}+\dfrac{\sqrt{17}}{4}$	**9** Write down both the solutions.

Practice

3 Solve by completing the square.
- **a** $x^2-4x-3=0$
- **b** $x^2-10x+4=0$
- **c** $x^2+8x-5=0$
- **d** $x^2-2x-6=0$
- **e** $2x^2+8x-5=0$
- **f** $5x^2+3x-4=0$

4 Solve by completing the square.
- **a** $(x-4)(x+2)=5$
- **b** $2x^2+6x-7=0$
- **c** $x^2-5x+3=0$

Hint

Get all terms onto one side of the

Solving quadratic equations by using the formula

Key points

- Any quadratic equation of the form $ax^2 + bx + c = 0$ can be solved using the formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- If $b^2 - 4ac$ is negative then the quadratic equation does not have any real solutions.
- It is useful to write down the formula before substituting the values for a, b and c.

Examples

Example 7   Solve $x^2 + 6x + 4 = 0$. Give your solutions in surd form.

$a = 1,\ b = 6,\ c = 4$ $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	**1** Identify a, b and c and write down the formula. Remember that $-b \pm \sqrt{b^2 - 4ac}$ is all over $2a$, not just part of it.
$x = \dfrac{-6 \pm \sqrt{6^2 - 4(1)(4)}}{2(1)}$	**2** Substitute $a = 1$, $b = 6$, $c = 4$ into the formula.
$x = \dfrac{-6 \pm \sqrt{20}}{2}$	**3** Simplify. The denominator is 2, but this is only because $a = 1$. The denominator will not always be 2.
$x = \dfrac{-6 \pm 2\sqrt{5}}{2}$	**4** Simplify $\sqrt{20}$. $\sqrt{20} = \sqrt{4 \times 5} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$
$x = -3 \pm \sqrt{5}$	**5** Simplify by dividing numerator and denominator by 2.
So $x = -3 - \sqrt{5}$ or $x = \sqrt{5} - 3$	**6** Write down both the solutions.

Example 8   Solve $3x^2 - 7x - 2 = 0$. Give your solutions in surd form.

$a = 3$, $b = -7$, $c = -2$ $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	1 Identify a, b and c, making sure you get the signs right and write down the formula. Remember that $-b \pm \sqrt{b^2 - 4ac}$ is all over $2a$, not just part of it.
$x = \dfrac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-2)}}{2(3)}$	2 Substitute $a = 3$, $b = -7$, $c = -2$ into the formula.
$x = \dfrac{7 \pm \sqrt{73}}{6}$ So $x = \dfrac{7 - \sqrt{73}}{6}$ or $x = \dfrac{7 + \sqrt{73}}{6}$	3 Simplify. The denominator is 6 when $a = 3$. A common mistake is to always write a denominator of 2. 4 Write down both the solutions.

Practice

5   Solve, giving your solutions in surd form.
   a $3x^2 + 6x + 2 = 0$ **b** $2x^2 - 4x - 7 = 0$

6   Solve the equation $x^2 - 7x + 2 = 0$

   Give your solutions in the form $\dfrac{a \pm \sqrt{b}}{c}$, where a, b and c are integers.

7   Solve $10x^2 + 3x + 3 = 5$
   Give your solution in surd form.

> **Hint**
>
> Get all terms onto one side of the equation.

Extend

8   Choose an appropriate method to solve each quadratic equation, giving your answer in surd form when necessary.
   a $4x(x - 1) = 3x - 2$
   b $10 = (x + 1)^2$
   c $x(3x - 1) = 10$

Solving linear simultaneous equations using the elimination method

Key points

- Two equations are simultaneous when they are both true at the same time.
- Solving simultaneous linear equations in two unknowns involves finding the value of each unknown which works for both equations.
- Make sure that the coefficient of one of the unknowns is the same in both equations.
- Eliminate this equal unknown by either subtracting or adding the two equations.

Examples

Example 1   Solve the simultaneous equations $3x + y = 5$ and $x + y = 1$

$3x + y = 5$ $-\quad x + y = 1$ $\overline{\quad 2x \qquad = 4}$ So $x = 2$	**1** Subtract the second equation from the first equation to eliminate the y term.
Using $x + y = 1$ $\quad\quad 2 + y = 1$ So $y = -1$	**2** To find the value of y, substitute $x = 2$ into one of the original equations.

Check:
   equation 1: $3 \times 2 + (-1) = 5$ YES
   equation 2: $2 + (-1) = 1$ YES
	3 Substitute the values of x and y into both equations to check your answers.

Example 2   Solve $x + 2y = 13$ and $5x - 2y = 5$ simultaneously.

$x + 2y = 13$ $+\quad 5x - 2y = \ 5$ $\overline{\quad 6x \qquad = 18}$ So $x = 3$	**1** Add the two equations together to eliminate the y term.
Using $x + 2y = 13$ $\quad\quad 3 + 2y = 13$ So $y = 5$	**2** To find the value of y, substitute $x = 3$ into one of the original equations.

Check:
   equation 1: $3 + 2 \times 5 = 13$ YES
   equation 2: $5 \times 3 - 2 \times 5 = 5$ YES
	3 Substitute the values of x and y into both equations to check your answers.

Example 3 Solve $2x + 3y = 2$ and $5x + 4y = 12$ simultaneously.

$(2x + 3y = 2) \times 4 \;\rightarrow\quad 8x + 12y = 8$ $(5x + 4y = 12) \times 3 \rightarrow\quad \underline{15x + 12y = 36}$ $\qquad\qquad\qquad\qquad\quad 7x \qquad = 28$ So $x = 4$ Using $\;2x \;+\; 3y \;= 2$ $\qquad\quad 2 \times 4 + 3y = 2$ So $y = -2$ Check: $\quad$ equation 1: $2 \times 4 + 3 \times (-2) = 2\quad$ YES $\quad$ equation 2: $5 \times 4 + 4 \times (-2) = 12\;$ YES	**1** Multiply the first equation by 4 and the second equation by 3 to make the coefficient of y the same for both equations. Then subtract the first equation from the second equation to eliminate the y term. **2** To find the value of y, substitute $x = 4$ into one of the original equations. **3** Substitute the values of x and y into both equations to check your answers.

Practice

Solve these simultaneous equations.

1 $4x + y = 8$
$\quad\;\;x + y = 5$

2 $3x + y = 7$
$\quad\;\;3x + 2y = 5$

3 $4x + y = 3$
$\quad\;\;3x - y = 11$

4 $3x + 4y = 7$
$\quad\;\;x - 4y = 5$

5 $2x + y = 11$
$\quad\;\;x - 3y = 9$

6 $2x + 3y = 11$
$\quad\;\;3x + 2y = 4$

Solving linear simultaneous equations using the substitution method

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A LEVEL LINKS
Scheme of work: 1c. Equations – quadratic/linear simultaneous
Textbook: Pure Year 1, 3.1 Linear simultaneous equations

Key points

- The subsitution method is the method most commonly used for A level. This is because it is the method used to solve linear and quadratic simultaneous equations.

Examples

Example 4   Solve the simultaneous equations $y = 2x + 1$ and $5x + 3y = 14$

$5x + 3(2x + 1) = 14$	**1** Substitute $2x + 1$ for y into the second equation.
$5x + 6x + 3 = 14$	**2** Expand the brackets and simplify.
$11x + 3 = 14$	
$11x = 11$	**3** Work out the value of x.
So $x = 1$	
Using $y = 2x + 1$	**4** To find the value of y, substitute
$y = 2 \times 1 + 1$	$x = 1$ into one of the original
So $y = 3$	equations.
Check:	**5** Substitute the values of x and y into
equation 1: $3 = 2 \times 1 + 1$ YES	both equations to check your
equation 2: $5 \times 1 + 3 \times 3 = 14$ YES	answers.

Example 5   Solve $2x - y = 16$ and $4x + 3y = -3$ simultaneously.

$y = 2x - 16$	**1** Rearrange the first equation.
$4x + 3(2x - 16) = -3$	**2** Substitute $2x - 16$ for y into the second equation.
$4x + 6x - 48 = -3$	**3** Expand the brackets and simplify.
$10x - 48 = -3$	
$10x = 45$	**4** Work out the value of x.
So $x = 4\frac{1}{2}$	
Using $y = 2x - 16$	**5** To find the value of y, substitute
$y = 2 \times 4\frac{1}{2} - 16$	$x = 4\frac{1}{2}$ into one of the original
So $y = -7$	equations.
Check:	**6** Substitute the values of x and y into
equation 1: $2 \times 4\frac{1}{2} - (-7) = 16$ YES	both equations to check your
equation 2: $4 \times 4\frac{1}{2} + 3 \times (-7) = -3$ YES	answers.

Practice

Solve these simultaneous equations.

7 $y = x - 4$
  $2x + 5y = 43$

8 $y = 2x - 3$
  $5x - 3y = 11$

9 $2y = 4x + 5$
  $9x + 5y = 22$

10 $2x = y - 2$
  $8x - 5y = -11$

11 $3x + 4y = 8$
  $2x - y = -13$

12 $3y = 4x - 7$
  $2y = 3x - 4$

13 $3x = y - 1$
  $2y - 2x = 3$

14 $3x + 2y + 1 = 0$
  $4y = 8 - x$

Extend

15 Solve the simultaneous equations $3x + 5y - 20 = 0$ and $2(x + y) = \dfrac{3(y - x)}{4}$.

Solving linear and quadratic simultaneous equations

Key points

- Make one of the unknowns the subject of the linear equation (rearranging where necessary).
- Use the linear equation to substitute into the quadratic equation.
- There are usually two pairs of solutions.

Examples

Example 1   Solve the simultaneous equations $y = x + 1$ and $x^2 + y^2 = 13$

$x^2 + (x + 1)^2 = 13$	**1** Substitute $x + 1$ for y into the second equation.
$x^2 + x^2 + x + x + 1 = 13$	**2** Expand the brackets and simplify.
$2x^2 + 2x + 1 = 13$	
$2x^2 + 2x - 12 = 0$	**3** Factorise the quadratic equation.
$(2x - 4)(x + 3) = 0$	
So $x = 2$ or $x = -3$	**4** Work out the values of x.
Using $y = x + 1$	**5** To find the value of y, substitute both values of x into one of the original equations.
When $x = 2$, $y = 2 + 1 = 3$	
When $x = -3$, $y = -3 + 1 = -2$	
So the solutions are	
$\quad x = 2,\ y = 3$ and $x = -3, y = -2$	
Check:	**6** Substitute both pairs of values of x and y into both equations to check your answers.
equation 1: $3 = 2 + 1$ YES	
$\quad\quad$ and $-2 = -3 + 1$ YES	
equation 2: $2^2 + 3^2 = 13$ YES	
$\quad\quad$ and $(-3)^2 + (-2)^2 = 13$ YES	

Example 2   Solve $2x + 3y = 5$ and $2y^2 + xy = 12$ simultaneously.

$x = \dfrac{5-3y}{2}$	**1** Rearrange the first equation.
$2y^2 + \left(\dfrac{5-3y}{2}\right) y = 12$	**2** Substitute $\dfrac{5-3y}{2}$ for x into the second equation. Notice how it is easier to substitute for x than for y.
$2y^2 + \dfrac{5y-3y^2}{2} = 12$	**3** Expand the brackets and simplify.
$4y^2 + 5y - 3y^2 = 24$	
$y^2 + 5y - 24 = 0$	
$(y + 8)(y - 3) = 0$	**4** Factorise the quadratic equation.
So $y = -8$ or $y = 3$	**5** Work out the values of y.
Using $2x + 3y = 5$	**6** To find the value of x, substitute both values of y into one of the original equations.
When $y = -8$, $2x + 3 \times (-8) = 5$, $x = 14.5$	
When $y = 3$, $2x + 3 \times 3 = 5$, $x = -2$	
So the solutions are	
$x = 14.5$, $y = -8$ and $x = -2$, $y = 3$	
Check:	**7** Substitute both pairs of values of x and y into both equations to check your answers.
equation 1: $2 \times 14.5 + 3 \times (-8) = 5$ YES	
and $2 \times (-2) + 3 \times 3 = 5$ YES	
equation 2: $2 \times (-8)^2 + 14.5 \times (-8) = 12$ YES	
and $2 \times (3)^2 + (-2) \times 3 = 12$ YES	

Practice

Solve these simultaneous equations.

1 $y = 2x + 1$
   $x^2 + y^2 = 10$

2 $y = 6 - x$
   $x^2 + y^2 = 20$

3 $y = x - 3$
   $x^2 + y^2 = 5$

4 $y = 9 - 2x$
   $x^2 + y^2 = 17$

5 $y = 3x - 5$
   $y = x^2 - 2x + 1$

6 $y = x - 5$
   $y = x^2 - 5x - 12$

7 $y = x + 5$
   $x^2 + y^2 = 25$

8 $y = 2x - 1$
   $x^2 + xy = 24$

9 $y = 2x$
   $y^2 - xy = 8$

10 $2x + y = 11$
   $xy = 15$

Extend

11 $x - y = 1$
    $x^2 + y^2 = 3$

12 $y - x = 2$
    $x^2 + xy = 3$

Quadratic inequalities

A LEVEL LINKS
Scheme of work: 1d. Inequalities – linear and quadratic (including graphical solutions)

Key points

- First replace the inequality sign by = and solve the quadratic equation.
- Sketch the graph of the quadratic function.
- Use the graph to find the values which satisfy the quadratic inequality.

Examples

Example 1   Find the set of values of x which satisfy $x^2 + 5x + 6 > 0$

$x^2 + 5x + 6 = 0$ $(x + 3)(x + 2) = 0$ $x = -3$ or $x = -2$	**1** Solve the quadratic equation by factorising.
It is above the x-axis where $x^2 + 5x + 6 > 0$ This part of the graph is not needed as this is where $x^2 + 5x + 6 < 0$	**2** Sketch the graph of $y = (x + 3)(x + 2)$ **3** Identify on the graph where $x^2 + 5x + 6 > 0$, i.e. where $y > 0$
$x < -3$ or $x > -2$	**4** Write down the values which satisfy the inequality $x^2 + 5x + 6 > 0$

Example 2   Find the set of values of x which satisfy $x^2 - 5x \leq 0$

$x^2 - 5x = 0$ $x(x - 5) = 0$ $x = 0$ or $x = 5$ 	**1** Solve the quadratic equation by factorising. **2** Sketch the graph of $y = x(x - 5)$ **3** Identify on the graph where $x^2 - 5x \leq 0$, i.e. where $y \leq 0$
$0 \leq x \leq 5$	**4** Write down the values which satisfy the inequality $x^2 - 5x \leq 0$

Example 3   Find the set of values of x which satisfy $-x^2 - 3x + 10 \geq 0$

$-x^2 - 3x + 10 = 0$ $(-x + 2)(x + 5) = 0$ $x = 2$ or $x = -5$	**1** Solve the quadratic equation by factorising.
	2 Sketch the graph of $y = (-x + 2)(x + 5) = 0$
	3 Identify on the graph where $-x^2 - 3x + 10 \geq 0$, i.e. where $y \geq 0$
$-5 \leq x \leq 2$	**3** Write down the values which satisfy the inequality $-x^2 - 3x + 10 \geq 0$

Practice

1 Find the set of values of x for which $(x + 7)(x - 4) \leq 0$

2 Find the set of values of x for which $x^2 - 4x - 12 \geq 0$

3 Find the set of values of x for which $2x^2 - 7x + 3 < 0$

4 Find the set of values of x for which $4x^2 + 4x - 3 > 0$

5 Find the set of values of x for which $12 + x - x^2 \geq 0$

Extend

Find the set of values which satisfy the following inequalities.

6 $x^2 + x \leq 6$

7 $x(2x - 9) < -10$

8 $6x^2 \geq 15 + x$

Straight line graphs

> **A LEVEL LINKS**
> **Scheme of work:** 2a. Straight-line graphs, parallel/perpendicular, length and area problems

Key points

- A straight line has the equation $y = mx + c$, where m is the gradient and c is the y-intercept (where $x = 0$).
- The equation of a straight line can be written in the form $ax + by + c = 0$, where a, b and c are integers.
- When given the coordinates (x_1, y_1) and (x_2, y_2) of two points on a line the gradient is calculated using the formula $m = \dfrac{y_2 - y_1}{x_2 - x_1}$
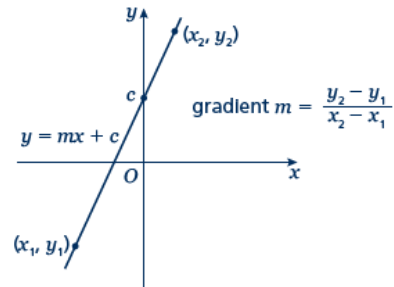
Examples

Example 1   A straight line has gradient $-\dfrac{1}{2}$ and y-intercept 3.

Write the equation of the line in the form $ax + by + c = 0$.

$m = -\dfrac{1}{2}$ and $c = 3$ So $y = -\dfrac{1}{2}x + 3$ $\dfrac{1}{2}x + y - 3 = 0$ $x + 2y - 6 = 0$	**1** A straight line has equation $y = mx + c$. Substitute the gradient and y-intercept given in the question into this equation. **2** Rearrange the equation so all the terms are on one side and 0 is on the other side. **3** Multiply both sides by 2 to eliminate the denominator.

Example 2   Find the gradient and the y-intercept of the line with the equation $3y - 2x + 4 = 0$.

$3y - 2x + 4 = 0$ $3y = 2x - 4$ $y = \dfrac{2}{3}x - \dfrac{4}{3}$ Gradient $= m = \dfrac{2}{3}$ y-intercept $= c = -\dfrac{4}{3}$	**1** Make y the subject of the equation. **2** Divide all the terms by three to get the equation in the form $y = \ldots$ **3** In the form $y = mx + c$, the gradient is m and the y-intercept is c.

Example 3 Find the equation of the line which passes through the point (5, 13) and has gradient 3.

$m = 3$ $y = 3x + c$ $13 = 3 \times 5 + c$ $13 = 15 + c$ $c = -2$ $y = 3x - 2$	**1** Substitute the gradient given in the question into the equation of a straight line $y = mx + c$. **2** Substitute the coordinates $x = 5$ and $y = 13$ into the equation. **3** Simplify and solve the equation. **4** Substitute $c = -2$ into the equation $y = 3x + c$

Example 4 Find the equation of the line passing through the points with coordinates (2, 4) and (8, 7).

$x_1 = 2$, $x_2 = 8$, $y_1 = 4$ and $y_2 = 7$ $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{7 - 4}{8 - 2} = \dfrac{3}{6} = \dfrac{1}{2}$ $y = \dfrac{1}{2}x + c$ $4 = \dfrac{1}{2} \times 2 + c$ $c = 3$ $y = \dfrac{1}{2}x + 3$	**1** Substitute the coordinates into the equation $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ to work out the gradient of the line. **2** Substitute the gradient into the equation of a straight line $y = mx + c$. **3** Substitute the coordinates of either point into the equation. **4** Simplify and solve the equation. **5** Substitute $c = 3$ into the equation $y = \dfrac{1}{2}x + c$

Practice

1 Find the gradient and the *y*-intercept of the following equations.

a $y = 3x + 5$ 　　　　　**b** $y = -\dfrac{1}{2}x - 7$

c $2y = 4x - 3$ 　　　　　**d** $x + y = 5$

e $2x - 3y - 7 = 0$ 　　　**f** $5x + y - 4 = 0$

> **Hint**
> Rearrange the equations
> to the form $y = mx + c$

2 Copy and complete the table, giving the equation of the line in the form $y = mx + c$.

Gradient	*y*-intercept	Equation of the line
5	0	
−3	2	
4	−7	

3 Find, in the form $ax + by + c = 0$ where a, b and c are integers, an equation for each of the lines with the following gradients and y-intercepts.

    a   gradient $-\dfrac{1}{2}$, y-intercept -7 **b**   gradient 2, y-intercept 0

    c   gradient $\dfrac{2}{3}$, y-intercept 4   **d**   gradient -1.2, y-intercept -2

4 Write an equation for the line which passes though the point $(2, 5)$ and has gradient 4.

5 Write an equation for the line which passes through the point $(6, 3)$ and has gradient $-\dfrac{2}{3}$

6 Write an equation for the line passing through each of the following pairs of points.
    a $(4, 5)$, $(10, 17)$ **b** $(0, 6)$, $(-4, 8)$
    c $(-1, -7)$, $(5, 23)$ **d** $(3, 10)$, $(4, 7)$

Extend

7 The equation of a line is $2y + 3x - 6 = 0$.
Write as much information as possible about this line.

Parallel and perpendicular lines

Key points

- When lines are parallel they have the same gradient.
- A line perpendicular to the line with equation $y = mx + c$ has gradient $-\dfrac{1}{m}$.
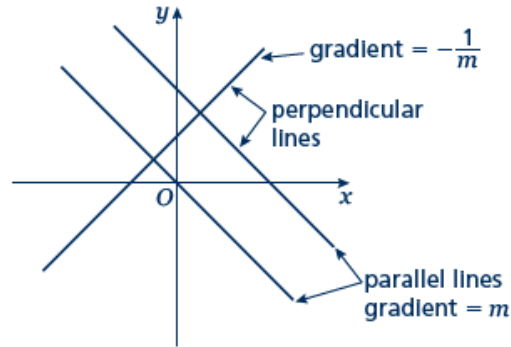
Examples

Example 1   Find the equation of the line parallel to $y = 2x + 4$ which passes through the point (4, 9).

$y = 2x + 4$ $m = 2$ $y = 2x + c$	**1** As the lines are parallel they have the same gradient. **2** Substitute $m = 2$ into the equation of a straight line $y = mx + c$.
$9 = 2 \times 4 + c$	**3** Substitute the coordinates into the equation $y = 2x + c$
$9 = 8 + c$ $c = 1$ $y = 2x + 1$	**4** Simplify and solve the equation. **5** Substitute $c = 1$ into the equation $y = 2x + c$

Example 2   Find the equation of the line perpendicular to $y = 2x - 3$ which passes through the point $(-2, 5)$.

$y = 2x - 3$ $m = 2$ $-\dfrac{1}{m} = -\dfrac{1}{2}$	**1** As the lines are perpendicular, the gradient of the perpendicular line is $-\dfrac{1}{m}$.
$y = -\dfrac{1}{2}x + c$	**2** Substitute $m = -\dfrac{1}{2}$ into $y = mx + c$.
$5 = -\dfrac{1}{2} \times (-2) + c$	**3** Substitute the coordinates $(-2, 5)$ into the equation $y = -\dfrac{1}{2}x + c$
$5 = 1 + c$ $c = 4$	**4** Simplify and solve the equation.
$y = -\dfrac{1}{2}x + 4$	**5** Substitute $c = 4$ into $y = -\dfrac{1}{2}x + c$.

Example 3   A line passes through the points $(0, 5)$ and $(9, -1)$.
Find the equation of the line which is perpendicular to the line and passes through its midpoint.

$x_1 = 0$, $x_2 = 9$, $y_1 = 5$ and $y_2 = -1$ $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-1-5}{9-0}$ $\qquad = \dfrac{-6}{9} = -\dfrac{2}{3}$ $-\dfrac{1}{m} = \dfrac{3}{2}$ $y = \dfrac{3}{2}x + c$ Midpoint $= \left(\dfrac{0+9}{2}, \dfrac{5+(-1)}{2}\right) = \left(\dfrac{9}{2}, 2\right)$ $2 = \dfrac{3}{2} \times \dfrac{9}{2} + c$ $c = -\dfrac{19}{4}$ $y = \dfrac{3}{2}x - \dfrac{19}{4}$	**1** Substitute the coordinates into the equation $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ to work out the gradient of the line. **2** As the lines are perpendicular, the gradient of the perpendicular line is $-\dfrac{1}{m}$. **3** Substitute the gradient into the equation $y = mx + c$. **4** Work out the coordinates of the midpoint of the line. **5** Substitute the coordinates of the midpoint into the equation. **6** Simplify and solve the equation. **7** Substitute $c = -\dfrac{19}{4}$ into the equation $y = \dfrac{3}{2}x + c$.

Practice

1 Find the equation of the line parallel to each of the given lines and which passes through each of the given points.

   a $y = 3x + 1$ $(3, 2)$ **b** $y = 3 - 2x$ $(1, 3)$

   c $2x + 4y + 3 = 0$ $(6, -3)$ **d** $2y - 3x + 2 = 0$ $(8, 20)$

2 Find the equation of the line perpendicular to $y = \dfrac{1}{2}x - 3$ which passes through the point $(-5, 3)$.

> **Hint**
>
> If $m = \dfrac{a}{b}$ then the negative reciprocal $-\dfrac{1}{m} = -\dfrac{b}{a}$

3 Find the equation of the line perpendicular to each of the given lines and which passes through each of the given points.

   a $y = 2x - 6$ $(4, 0)$ **b** $y = -\dfrac{1}{3}x + \dfrac{1}{2}$ $(2, 13)$

   c $x - 4y - 4 = 0$ $(5, 15)$ **d** $5y + 2x - 5 = 0$ $(6, 7)$

4 In each case find an equation for the line passing through the origin which is also perpendicular to the line joining the two points given.

   a $(4, 3)$, $(-2, -9)$ **b** $(0, 3)$, $(-10, 8)$

Extend

5 Work out whether these pairs of lines are parallel, perpendicular or neither.

a $y = 2x + 3$
$y = 2x - 7$

b $y = 3x$
$2x + y - 3 = 0$

c $y = 4x - 3$
$4y + x = 2$

d $3x - y + 5 = 0$
$x + 3y = 1$

e $2x + 5y - 1 = 0$
$y = 2x + 7$

f $2x - y = 6$
$6x - 3y + 3 = 0$

6 The straight line $\mathbf{L_1}$ passes through the points A and B with coordinates $(-4, 4)$ and $(2, 1)$, respectively.

a Find the equation of $\mathbf{L_1}$ in the form $ax + by + c = 0$

The line $\mathbf{L_2}$ is parallel to the line $\mathbf{L_1}$ and passes through the point C with coordinates $(-8, 3)$.

b Find the equation of $\mathbf{L_2}$ in the form $ax + by + c = 0$

The line $\mathbf{L_3}$ is perpendicular to the line $\mathbf{L_1}$ and passes through the origin.

c Find an equation of $\mathbf{L_3}$

Answers

1 **a** $6x - 3$ **b** $-10pq - 8q^2$

    c $-3xy + 2y^2$

2 **a** $21x + 35 + 12x - 48 = 33x - 13$

    b $40p - 16 - 12p - 27 = 28p - 43$

    c $27s + 9 - 30s + 50 = -3s + 59 = 59 - 3s$

    d $8x - 6 - 3x - 5 = 5x - 11$

3 **a** $12x^2 + 24x$ **b** $20k^3 - 48k$

    c $10h - 12h^3 - 22h^2$ **d** $21s^2 - 21s^3 - 6s$

4 **a** $-y^2 - 4$ **b** $5x^2 - 11x$

    c $2p - 7p^2$ **d** $6b^2$

5 $y - 4$

6 **a** $-1 - 2m$ **b** $5p^3 + 12p^2 + 27p$

7 $7x(3x - 5) = 21x^2 - 35x$

8 **a** $x^2 + 9x + 20$ **b** $x^2 + 10x + 21$

    c $x^2 + 5x - 14$ **d** $x^2 - 25$

    e $2x^2 + x - 3$ **f** $6x^2 - x - 2$

    g $10x^2 - 31x + 15$ **h** $12x^2 + 13x - 14$

    i $18x^2 + 39xy + 20y^2$ **j** $x^2 + 10x + 25$

    k $4x^2 - 28x + 49$ **l** $16x^2 - 24xy + 9y^2$

9 $2x^2 - 2x + 25$

10 **a** $x^2 - 1 - \dfrac{2}{x^2}$ **b** $x^2 + 2 + \dfrac{1}{x^2}$

Answers

1
  a $3\sqrt{5}$
              b $5\sqrt{5}$

  c $4\sqrt{3}$
              d $5\sqrt{7}$

  e $10\sqrt{3}$
             f $2\sqrt{7}$

  g $6\sqrt{2}$
             h $9\sqrt{2}$

2
  a $15\sqrt{2}$
          b $\sqrt{5}$

  c $3\sqrt{2}$
           d $\sqrt{3}$

  e $6\sqrt{7}$
           f $5\sqrt{3}$

3
  a -1
            b $9-\sqrt{3}$

  c $10\sqrt{5}-7$
       d $26-4\sqrt{2}$

4
  a $\dfrac{\sqrt{5}}{5}$
           b $\dfrac{\sqrt{11}}{11}$

  c $\dfrac{2\sqrt{7}}{7}$
         d $\dfrac{\sqrt{2}}{2}$

  e $\sqrt{2}$
            f $\sqrt{5}$

  g $\dfrac{\sqrt{3}}{3}$
          h $\dfrac{1}{3}$

5
  a $\dfrac{3+\sqrt{5}}{4}$
     b $\dfrac{2(4-\sqrt{3})}{13}$
     c $\dfrac{6(5+\sqrt{2})}{23}$

6 $x-y$

7
  a $3+2\sqrt{2}$
        b $\dfrac{\sqrt{x}+\sqrt{y}}{x-y}$

Answers

1 **a** 1 **b** 1 **c** 1 **d** 1

2 **a** 7 **b** 4 **c** 5 **d** 2

3 **a** 125 **b** 32 **c** 343 **d** 8

4 **a** $\dfrac{1}{25}$ **b** $\dfrac{1}{64}$ **c** $\dfrac{1}{32}$ **d** $\dfrac{1}{36}$

5 **a** $\dfrac{3x^3}{2}$ **b** $5x^2$

    c $3x$ **d** $\dfrac{y}{2x^2}$

    e $y^{\frac{1}{2}}$ **f** c^{-3}

    g $2x^6$ **h** x

6 **a** $\dfrac{1}{2}$ **b** $\dfrac{1}{9}$ **c** $\dfrac{8}{3}$

    d $\dfrac{1}{4}$ **e** $\dfrac{4}{3}$ **f** $\dfrac{16}{9}$

7 **a** x^{-1} **b** x^{-7} **c** $x^{\frac{1}{4}}$

    d $x^{\frac{2}{5}}$ **e** $x^{-\frac{1}{3}}$ **f** $x^{-\frac{2}{3}}$

8 **a** $\dfrac{1}{x^3}$ **b** 1 **c** $\sqrt[5]{x}$

    d $\sqrt[5]{x^2}$ **e** $\dfrac{1}{\sqrt{x}}$ **f** $\dfrac{1}{\sqrt[4]{x^3}}$

9 **a** $5x^{\frac{1}{2}}$ **b** $2x^{-3}$ **c** $\dfrac{1}{3}x^{-4}$

    d $2x^{-\frac{1}{2}}$ **e** $4x^{-\frac{1}{3}}$ **f** $3x^0$

10 **a** $x^3 + x^{-2}$ **b** $x^3 + x$ **c** $x^{-2} + x^{-7}$

Answers

1 **a** $2x^3y^3(3x - 5y)$ **b** $7a^3b^2(3b^3 + 5a^2)$

     c $5x^2y^2(5 - 2x + 3y)$

2 **a** $(x + 3)(x + 4)$ **b** $(x + 7)(x - 2)$

     c $(x - 5)(x - 6)$ **d** $(x - 8)(x + 3)$

     e $(x - 9)(x + 2)$ **f** $(x + 5)(x - 4)$

     g $(x - 8)(x + 5)$ **h** $(x + 7)(x - 4)$

3 **a** $(6x - 7y)(6x + 7y)$ **b** $(2x - 9y)(2x + 9y)$

     c $2(3a - 10bc)(3a + 10bc)$

4 **a** $(x - 1)(2x + 3)$ **b** $(3x + 1)(2x + 5)$

     c $(2x + 1)(x + 3)$ **d** $(3x - 1)(3x - 4)$

     e $(5x + 3)(2x + 3)$ **f** $2(3x - 2)(2x - 5)$

5 **a** $\dfrac{2(x+2)}{x-1}$ **b** $\dfrac{x}{x-1}$

     c $\dfrac{x+2}{x}$ **d** $\dfrac{x}{x+5}$

     e $\dfrac{x+3}{x}$ **f** $\dfrac{x}{x-5}$

6 **a** $\dfrac{3x+4}{x+7}$ **b** $\dfrac{2x+3}{3x-2}$

     c $\dfrac{2-5x}{2x-3}$ **d** $\dfrac{3x+1}{x+4}$

7 $(x + 5)$

8 $\dfrac{4(x+2)}{x-2}$

Answers

1 **a** $(x + 2)^2 - 1$ **b** $(x - 5)^2 - 28$

     c $(x - 4)^2 - 16$ **d** $(x + 3)^2 - 9$

     e $(x - 1)^2 + 6$ **f** $\left(x + \dfrac{3}{2}\right)^2 - \dfrac{17}{4}$

2 **a** $2(x - 2)^2 - 24$ **b** $4(x - 1)^2 - 20$

     c $3(x + 2)^2 - 21$ **d** $2\left(x + \dfrac{3}{2}\right)^2 - \dfrac{25}{2}$

3 **a** $2\left(x + \dfrac{3}{4}\right)^2 + \dfrac{39}{8}$ **b** $3\left(x - \dfrac{1}{3}\right)^2 - \dfrac{1}{3}$

     c $5\left(x + \dfrac{3}{10}\right)^2 - \dfrac{9}{20}$ **d** $3\left(x + \dfrac{5}{6}\right)^2 + \dfrac{11}{12}$

4 $(5x + 3)^2 + 3$

Answers

1 **a** $x = 0$ or $x = -\dfrac{2}{3}$ **b** $x = 0$ or $x = \dfrac{3}{4}$

    c $x = -5$ or $x = -2$ **d** $x = 2$ or $x = 3$

    e $x = -1$ or $x = 4$ **f** $x = -5$ or $x = 2$

    g $x = 4$ or $x = 6$ **h** $x = -6$ or $x = 6$

    i $x = -7$ or $x = 4$ **j** $x = 3$

    k $x = -\dfrac{1}{2}$ or $x = 4$ **l** $x = -\dfrac{2}{3}$ or $x = 5$

2 **a** $x = -2$ or $x = 5$ **b** $x = -1$ or $x = 3$

    c $x = -8$ or $x = 3$ **d** $x = -6$ or $x = 7$

    e $x = -5$ or $x = 5$ **f** $x = -4$ or $x = 7$

    g $x = -3$ or $x = 2\dfrac{1}{2}$ **h** $x = -\dfrac{1}{3}$ or $x = 2$

3 **a** $x = 2 + \sqrt{7}$ or $x = 2 - \sqrt{7}$ **b** $x = 5 + \sqrt{21}$ or $x = 5 - \sqrt{21}$

    c $x = -4 + \sqrt{21}$ or $x = -4 - \sqrt{21}$ **d** $x = 1 + \sqrt{7}$ or $x = 1 - \sqrt{7}$

    e $x = -2 + \sqrt{6.5}$ or $x = -2 - \sqrt{6.5}$ **f** $x = \dfrac{-3 + \sqrt{89}}{10}$ or $x = \dfrac{-3 - \sqrt{89}}{10}$

4 **a** $x = 1 + \sqrt{14}$ or $x = 1 - \sqrt{14}$ **b** $x = \dfrac{-3 + \sqrt{23}}{2}$ or $x = \dfrac{-3 - \sqrt{23}}{2}$

    c $x = \dfrac{5 + \sqrt{13}}{2}$ or $x = \dfrac{5 - \sqrt{13}}{2}$

5 **a** $x = -1 + \dfrac{\sqrt{3}}{3}$ or $x = -1 - \dfrac{\sqrt{3}}{3}$ **b** $x = 1 + \dfrac{3\sqrt{2}}{2}$ or $x = 1 - \dfrac{3\sqrt{2}}{2}$

6 $x = \dfrac{7 + \sqrt{41}}{2}$ or $x = \dfrac{7 - \sqrt{41}}{2}$

7 $x = \dfrac{-3 + \sqrt{89}}{20}$ or $x = \dfrac{-3 - \sqrt{89}}{20}$

8 **a** $x = \dfrac{7 + \sqrt{17}}{8}$ or $x = \dfrac{7 - \sqrt{17}}{8}$

    b $x = -1 + \sqrt{10}$ or $x = -1 - \sqrt{10}$

    c $x = -1\dfrac{2}{3}$ or $x = 2$

Answers

1     $x = 1, y = 4$

2     $x = 3, y = -2$

3     $x = 2, y = -5$

4     $x = 3, y = -\dfrac{1}{2}$

5     $x = 6, y = -1$

6     $x = -2, y = 5$

7     $x = 9, y = 5$

8     $x = -2, y = -7$

9     $x = \dfrac{1}{2}, y = 3\dfrac{1}{2}$

10    $x = \dfrac{1}{2}, y = 3$

11    $x = -4, y = 5$

12    $x = -2, y = -5$

13    $x = \dfrac{1}{4}, y = 1\dfrac{3}{4}$

14    $x = -2, y = 2\dfrac{1}{2}$

15    $x = -2\dfrac{1}{2}, y = 5\dfrac{1}{2}$

Answers

1    $x = 1, y = 3$

     $x = -\dfrac{9}{5}, y = -\dfrac{13}{5}$

2    $x = 2, y = 4$
     $x = 4, y = 2$

3    $x = 1, y = -2$
     $x = 2, y = -1$

4    $x = 4, y = 1$

     $x = \dfrac{16}{5}, y = \dfrac{13}{5}$

5    $x = 3, y = 4$
     $x = 2, y = 1$

6    $x = 7, y = 2$
     $x = -1, y = -6$

7    $x = 0, y = 5$
     $x = -5, y = 0$

8    $x = -\dfrac{8}{3}, y = -\dfrac{19}{3}$
     $x = 3, y = 5$

9    $x = -2, y = -4$
     $x = 2, y = 4$

10   $x = \dfrac{5}{2}, y = 6$
     $x = 3, y = 5$

11   $x = \dfrac{1 + \sqrt{5}}{2}, y = \dfrac{-1 + \sqrt{5}}{2}$

     $x = \dfrac{1 - \sqrt{5}}{2}, y = \dfrac{-1 - \sqrt{5}}{2}$

12   $x = \dfrac{-1 + \sqrt{7}}{2}, y = \dfrac{3 + \sqrt{7}}{2}$

     $x = \dfrac{-1 - \sqrt{7}}{2}, y = \dfrac{3 - \sqrt{7}}{2}$

Answers

1 $-7 \le x \le 4$

2 $x \le -2$ or $x \ge 6$

3 $\dfrac{1}{2} < x < 3$

4 $x < -\dfrac{3}{2}$ or $x > \dfrac{1}{2}$

5 $-3 \le x \le 4$

6 $-3 \le x \le 2$

7 $2 < x < 2\dfrac{1}{2}$

8 $x \le -\dfrac{3}{2}$ or $x \ge \dfrac{5}{3}$

Answers

1 **a** $m = 3, c = 5$ **b** $m = -\dfrac{1}{2}, c = -7$

    c $m = 2, c = -\dfrac{3}{2}$ **d** $m = -1, c = 5$

    e $m = \dfrac{2}{3}, c = -\dfrac{7}{3}$ or $-2\dfrac{1}{3}$ **f** $m = -5, c = 4$

2

Gradient	*y*-intercept	Equation of the line
5	0	$y = 5x$
–3	2	$y = -3x + 2$
4	–7	$y = 4x - 7$

3 **a** $x + 2y + 14 = 0$ **b** $2x - y = 0$

    c $2x - 3y + 12 = 0$ **d** $6x + 5y + 10 = 0$

4 $y = 4x - 3$

5 $y = -\dfrac{2}{3}x + 7$

6 **a** $y = 2x - 3$ **b** $y = -\dfrac{1}{2}x + 6$

    c $y = 5x - 2$ **d** $y = -3x + 19$

7 $y = -\dfrac{3}{2}x + 3$, the gradient is $-\dfrac{3}{2}$ and the *y*-intercept is 3.

The line intercepts the axes at $(0, 3)$ and $(2, 0)$.

Students may sketch the line or give coordinates that lie on the line such as $\left(1, \dfrac{3}{2}\right)$ or $(4, -3)$.

Answers

1 **a** $y = 3x - 7$ **b** $y = -2x + 5$

     c $y = -\dfrac{1}{2}x$ **d** $y = \dfrac{3}{2}x + 8$

2 $y = -2x - 7$

3 **a** $y = -\dfrac{1}{2}x + 2$ **b** $y = 3x + 7$

     c $y = -4x + 35$ **d** $y = \dfrac{5}{2}x - 8$

4 **a** $y = -\dfrac{1}{2}x$ **b** $y = 2x$

5 **a** Parallel   **b** Neither   **c** Perpendicular
     d Perpendicular   **e** Neither   **f** Parallel

6 **a** $x + 2y - 4 = 0$ **b** $x + 2y + 2 = 0$ **c** $y = 2x$